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Gluon polarization in nucleons*

A. Saalfeld, G. Piller, L. Mankiewicz[a]

Institut für Theoretische Physik, TU München, James-Franck-Strasse, D-85748 Garching, Germany

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Abstract. In QCD the gauge-invariant gluon polarization ΔG in a nucleon can be defined either in a non-local way as the integral over the Ioffe-time distribution of polarized gluons, or in light-cone gauge as the forward matrix element of the local topological current. We have investigated both possibilities within the framework of QCD sum rules. Although the topological current is built from local fields, we have found that its matrix element retains sensitivity to large longitudinal distances. Because QCD sum rules produce artificial oscillations of the Ioffe-time distribution of polarized glue at moderate and large light-like distances, the calculation of the matrix element of the topological current results in a small value of $\Delta G(\mu^2 \sim 1\,\mathrm{GeV}^2) \approx 0.6 \pm 0.2$. In a more consistent approach QCD sum rules are used to describe the polarized gluon distribution only at small light-like distances. Assuming that significant contributions to ΔG arise only from longitudinal length scales not larger than the nucleon size leads to $\Delta G(\mu^2 \sim 1\,\mathrm{GeV}^2) \approx 2 \pm 1$.

1 Introduction

Thanks to celebrated factorization theorems [1] hard scattering of highly virtual probes from nucleons can be characterized in QCD by universal, process independent, non-perturbative distribution functions which contain all relevant information about the long-distance dynamics of the target. At twist-2 these are identified in the framework of the QCD-improved parton model with scale-dependent quark and gluon light-cone distributions. For spin-1/2 targets they include unpolarized quark and gluon distributions and their polarized counterparts related to longitudinal and transverse target polarizations. Quantum numbers and chiral properties of the hard probe determine which particular set of distribution functions can be accessed in a specific process. Until now most information about the nucleon structure has been obtained from deep-inelastic lepton scattering experiments. In particular, recent measurements of polarized deep-inelastic scattering at CERN [2] and SLAC [3] have shown that a relatively small fraction of the nucleon spin is carried by quarks. This has started an ongoing debate about remaining contributions to the nucleon spin [4] which may result from gluon polarization and orbital angular momentum. Here especially the polarized gluon distribution $\Delta G(u, \mu^2)$[1] became of interest since it turned out to be measurable in future high-energy experiments, e.g. charm and direct photon production [5, 6].

At present the only available information about the magnitude of the total gluon polarization

$$\Delta G(\mu^2) = \int_0^1 du\, \Delta G(u, \mu^2) \tag{1}$$

results from the analysis of scaling violations in the polarized structure function $g_1(u, Q^2)$. Here a relatively large gluon polarization has been found even at a low normalization scale $\Delta G(\mu^2 = 1\mathrm{GeV}^2) = 1.6 \pm 0.9$ [7]. The main objective of the present paper is to discuss a framework in which one may determine $\Delta G(\mu^2)$ using presently known non-perturbative methods.

The link between parton model ideas and QCD is provided by the Operator Product Expansion (OPE) [1]. It allows to relate moments of parton distributions to matrix elements of local operators with appropriate quantum numbers[2]. The latter are computable either in some approximate way using e.g. model descriptions of hadronic structure [8,9] or QCD sum rules [10], or at least in principle, through lattice simulations [11].

An alternative, but completely equivalent picture views twist-2 parton distribution functions as normalized Fourier transforms of nucleon matrix elements of non-local QCD operators [12], constructed as gauge-invariant overlap of two quark or gluon fields separated by a light-like distance. Since the first few coefficients of the Taylor expansions of these non-local matrix elements around the origin are given by matrix elements of local, low-

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[a] On leave of absence from N. Copernicus Astronomical Center, Polish Academy of Science, ul. Bartycka 18, PL–00-716 Warsaw (Poland)

[1] Throughout this paper we denote the Bjorken variable x_{Bj} by u to avoid confusion with the space-time variable x

[2] We define the n-th moment of a distribution $F(u)$ as $\Gamma_n[F] = \int_0^1 du u^{n-1} F(u)$

dimensional twist-2 operators, the domain of small longitudinal distances is determined by known non-perturbative QCD methods. However for increasing longitudinal distances more and more twist-2 matrix elements are required and soon one enters a region where lattice QCD and other non-perturbative approaches are practically not applicable any more. Phenomenologically, for unpolarized charge conjugation even quark and gluon distributions one observes a transition between the region of small and large longitudinal distances around a length scale which corresponds to the electromagnetic nucleon size of around 2 fm [13]. Beyond that scale the matrix elements of the corresponding string operators become smooth and approximately flat as functions of longitudinal distance – at least at low normalization scales. The matrix elements of the string operators which correspond to C-odd unpolarized quark distributions decrease and become small at longitudinal distances beyond 2 fm. In both cases, however, the region of large longitudinal distances is beyond the scope of presently available non-perturbative methods, and one has to resort to approximations such as Regge theory. An ideal non-perturbative QCD observable should therefore avoid contributions from large longitudinal distances, i.e. it should be sensitive only to those degrees of freedom which reveal themselves at length scales which are not larger than the nucleon size.

Although the total gluon polarization receives most interest from a phenomenological point of view, establishing a framework for its evaluation has its own importance. This is due to the fact that there is no local, gauge invariant operator which can serve as a "gluon polarization partonometer", i.e. which yields a matrix element associated with $\Delta G(\mu^2)$. In general two different ways to define $\Delta G(\mu^2)$ have been discussed in the literature so far. It has been known for a long time that the gluon polarization can be related to the matrix element of a gauge invariant, but non-local gluonic string operator [14,15]:

$$O_{\rm NL} = n_\mu n_\nu \int_0^\infty d\lambda\, {\rm Tr}\, G^{\mu\xi}(\lambda n)[\lambda n;0]\tilde{G}_\xi{}^\nu(0). \qquad (2)$$

Here $G^{\mu\nu}$ and $\tilde{G}^{\mu\nu}$ denote the gluon field strength and its dual, respectively, and n_μ is a light-like vector with $n^2=0$, $n\cdot a = a^0 + a^3 \equiv a^+$ for any four-vector a. The trace in (2) is performed in color space and the path-ordered exponential in the adjoint representation $[\lambda n;0]$ guarantees gauge invariance. Note however that so far most experience has been obtained in dealing with matrix elements of local operators, and therefore it is not clear a'priori how one should apply the non-local operator (2) in practice. In our recent work [16] we have argued that a computation of $\Delta G(\mu^2)$ using the operator $O_{\rm NL}$ at some low normalization scale is possible, but requires an insight into the nature of contributions arising from large longitudinal distances. Once one accepts a point of view, supported e.g. by Regge theory, that despite its non-local character the gluon polarization $\Delta G(\mu^2)$ receives only minor contributions from large longitudinal distances, an approximate "gluon polarization partonometer" can be constructed in a gauge-invariant way. In its simplest version it takes into account information encoded in only two first computable QCD moments of the polarized gluon distribution function.

On the other hand in light-cone gauge $n\cdot A = A^+ = 0$ the operator $O_{\rm NL}$ assumes a local form, identical to the $n\cdot K = K^+$ component of the topological current [14,15]:

$$K_\mu = \frac{\alpha_s}{2\pi}\epsilon_{\mu\nu\rho\sigma}A_a^\nu\left(\partial^\rho A_a^\sigma + \frac{1}{3}g f_{abc}A_b^\rho A_c^\sigma\right). \qquad (3)$$

Consequently it is suggestive to use $n\cdot K$ as a gluon polarization partonometer due to its local character. Corresponding calculations in the framework of the bag model have been performed recently in [17]. Nevertheless, as we shall discuss, although formally the operator $n\cdot K$ is built from local fields, its matrix element is sensitive to large longitudinal distances in the same way as the matrix element of the non-local operator in (2). Thus the advantage of using $n\cdot K$ instead of $O_{\rm NL}$ is illusory. This is illustrated below within the framework of QCD sum rules. We find that our estimates for $\Delta G(\mu^2)$ as obtained from the gluon polarization partonometer presented in [16], and from the local operator (3) differ approximately by a factor of four. This discrepancy emphasizes, as we will show, the role of contributions from different longitudinal length scales.

The remainder of this paper is organized as follows: in Sect. 2 we collect the most important facts about the operator definition of the polarized gluon distribution and the gluon polarization integral. In Sect. 3 we present a QCD sum rule estimate for $\Delta G(\mu^2)$ starting out from the matrix element of $n\cdot K$ in light-cone gauge, and review an estimate for $\Delta G(\mu^2)$ using the gluon partonometer introduced in [16]. We then discuss and explain in Sect. 4 the reasons for the surprising discrepancy between the results obtained with these two methods. Finally Sect. 5 is devoted to a summary and conclusions.

2 Polarized gluon distribution in QCD

In QCD parton distributions can be related to matrix elements of twist-2 non-local operators [12]. In this framework unpolarized and polarized gluon distributions are defined through matrix elements of the light-cone string operators:

$$O_G(\Delta;0) = n_\mu n_\nu {\rm Tr}\, G^{\mu\xi}(\Delta)[\Delta;0]G_\xi{}^\nu(0), \qquad (4)$$
$$O_{\Delta G}(\Delta;0) = n_\mu n_\nu {\rm Tr}\, G^{\mu\xi}(\Delta)[\Delta;0]\tilde{G}_\xi{}^\nu(0). \qquad (5)$$

Here Δ stands for a light-like vector being proportional to n. The path-ordered exponential

$$[\Delta;0] = {\rm P}\exp\left[ig\Delta_\mu\int_0^1 d\lambda A^\mu(\Delta\lambda)\right], \qquad (6)$$

with the strong coupling constant g and the gluon field A^μ guarantees gauge invariance of the parton distributions (7,8). The forward matrix elements of the string operators (4,5) between nucleon states with momentum p and spin s define the unpolarized and polarized gluon distribution of a nucleon, $G(u,\mu^2)$ and $\Delta G(u,\mu^2)$, as a function of the Bjorken variable u and the normalization scale μ^2:

$$\begin{aligned}&\frac{1}{2}\sum_s \langle p,s|O_G(\Delta,0)|p,s\rangle_{\mu^2}\\&\quad=(p\cdot n)^2\int_0^1 du\, u\, G(u,\mu^2)\cos\left[u(p\cdot\Delta)\right], \qquad (7)\\&\langle p,s|O_{\Delta G}(\Delta,0)|p,s\rangle_{\mu^2}\\&\quad=(p\cdot n)(s\cdot n)\int_0^1 du\, u\,\Delta G(u,\mu^2)\sin\left[u(p\cdot\Delta)\right]. \qquad (8)\end{aligned}$$

The invariant measure of the light-cone distance between the two gluon fields in (7,8) is given by the so called Ioffe-time $z=p\cdot\Delta$. Furthermore note that for a target polarized in the 3-direction one has $s\cdot n=p\cdot n$. Taking the Fourier transform of (7,8) yields the distribution functions:

$$\begin{aligned}u\,G(u,\mu^2)&=\frac{1}{\pi\,(p\cdot n)^2}\\&\quad\times\int_0^\infty dz\sum_s\langle p,s|O_G(\Delta,0)|p,s\rangle_{\mu^2}\cos(uz), \qquad (9)\\u\,\Delta G(u,\mu^2)&=\frac{2}{\pi\,(p\cdot n)(s\cdot n)}\\&\quad\times\int_0^\infty dz\,\langle p,s|O_{\Delta G}(\Delta,0)|p,s\rangle_{\mu^2}\sin(uz). \qquad (10)\end{aligned}$$

These definitions are of course in agreement with the perceptions of the parton model. Indeed in light-cone gauge, $n\cdot A=0$, one can express the distributions (9,10) in terms of right- and left-handed gluon operators defined as $G^{+R(L)}=\varepsilon_\mu^{R(L)}G^{+\mu}$, with the polarization vectors $\varepsilon_R^\mu=(0,-1,-i,0)/\sqrt{2}$ and $\varepsilon_L^\mu=(0,1,-i,0)/\sqrt{2}$:

$$\begin{aligned}u\,G(u,\mu^2)&=\frac{1}{\pi\,p\cdot n}\int_0^\infty d\lambda\,\cos\left(p\cdot n\,\lambda u\right)\\&\quad\times\sum_s Tr\langle p,s|\left(G^{+R}(n\lambda)\right)^\dagger G^{+R}(0)\\&\quad+\left(G^{+L}(n\lambda)\right)^\dagger G^{+L}(0)|p,s\rangle_{\mu^2}, \qquad (11)\\u\,\Delta G(u,\mu^2)&=\frac{2i}{\pi\,s\cdot n}\int_0^\infty d\lambda\,\sin\left(p\cdot n\,\lambda u\right)\\&\quad\times Tr\langle p,s|\left(G^{+R}(n\lambda)\right)^\dagger G^{+R}(0)\\&\quad-\left(G^{+L}(n\lambda)\right)^\dagger G^{+L}(0)|p,s\rangle_{\mu^2}. \qquad (12)\end{aligned}$$

After rewriting the gluon field strength tensor in terms of light-cone quantized fields one obtains:

$$\begin{aligned}u\,G(u,\mu^2)&=\int dx\,d^2k_\perp\delta(u-x)x\\&\quad\times\left[n_g(x,\mathbf{k}_\perp,R,\mu^2)+n_g(x,\mathbf{k}_\perp,L,\mu^2)\right], \qquad (13)\\u\,\Delta G(u,\mu^2)&=\int dx\,d^2k_\perp\delta(u-x)x\\&\quad\times\left[n_g(x,\mathbf{k}_\perp,R,\mu^2)-n_g(x,\mathbf{k}_\perp,L,\mu^2)\right], \qquad (14)\end{aligned}$$

where $n_g(x,\mathbf{k}_\perp,R(L),\mu^2)$ denotes the light-cone distribution function of right- (left-) handed gluons with light-cone momentum fraction x and transverse momentum $\mathbf{k}_\perp$ [14].

In the following we focus on the polarized gluon distribution. Performing a Taylor expansion of (8) around $\Delta=0$ leads to well-known relations, or sum rules, between the moments of the polarized gluon distribution $\Delta G(u,\mu^2)$ and nucleon matrix elements of local QCD operators [18]:

$$\begin{aligned}&\int_0^1 du\,u^{l-1}\,\Delta G(u,\mu^2)\equiv\Gamma_l(\mu^2),\quad\text{with } l=3,5,\dots,\\&n_\mu n_\nu\mathrm{Tr}\,\langle p,s|G^{\mu\xi}(0)(in\cdot D)^{l-2}\tilde{G}_\xi{}^\nu(0)|p,s\rangle\\&\quad=(s\cdot n)(p\cdot n)^l\,\Gamma_l(\mu^2). \qquad (15)\end{aligned}$$

Note, however, that a sum rule for $l=1$ which corresponds to the integrated gluon polarization $\Delta G(\mu^2)$ is lacking. This is due to the fact that a suitable gauge invariant, charge conjugation even, local operator which may serve as a gluon polarization partonometer does not exist.

On the other hand we find from (8) that the polarized gluon distribution $\Delta G(\mu^2)$ is determined by an integral over the corresponding Ioffe-time distribution which is defined as:

$$\Gamma(z,\mu^2)=\int_0^1 du\,u\Delta G(u,\mu^2)\sin(uz). \qquad (16)$$

Since in the distribution sense one has:

$$\int_0^\infty dz\sin(uz)=\frac{1}{2}\left(\frac{1}{u+i\epsilon}+\frac{1}{u-i\epsilon}\right)=PV\frac{1}{u}, \qquad (17)$$

where PV denotes the principal value prescription, we indeed obtain:

$$\Delta G(\mu^2)=\int_0^\infty dz\,\Gamma(z,\mu^2)\,. \qquad (18)$$

It is important to realize that $\Gamma(z,\mu^2)\sim z^{\alpha-2}$ for large z, if $\Delta G(u,\mu^2)\sim u^{-\alpha}$ at small u. Therefore as long as $\alpha<1$ the integral over the polarized gluon distribution (18) exists as it converges at large z in an absolute sense. (At small z the integrand in (18) should not cause harm since there $\Gamma(z)\approx z\,\Gamma_3$, with an anticipated finite third moment Γ_3.) Expanding the RHS of (16) around $z=0$ yields a Taylor expansion of the Ioffe distribution $\Gamma(z,\mu^2)$ with coefficients proportional to the odd moments $\Gamma_l(\mu^2)$:

$$\begin{aligned}\Gamma(z,\mu^2)&=\Gamma_3(\mu^2)\,z-\frac{1}{6}\Gamma_5(\mu^2)\,z^3\\&\quad+\frac{1}{120}\Gamma_7(\mu^2)\,z^5-\dots \qquad (19)\end{aligned}$$

Since each of these moments can be calculated, at least formally, as a reduced matrix element of a local, gauge invariant operator, the convergent integral (18) and hence $\Delta G(\mu^2)$ itself is a gauge-invariant quantity.

As already mentioned in the introduction, in light-cone gauge the gluon polarization $\Delta G(\mu^2)$ can also be related to the expectation value of the topological current (3):

$$\langle p,s|n\cdot K(0)|p,s\rangle_{\mu^2} = (s\cdot n)\frac{\alpha_s}{\pi}\int_0^\infty dz\, \Gamma(z,\mu^2) = (s\cdot n)\frac{\alpha_s}{\pi}\Delta G(\mu^2). \quad (20)$$

Indeed, in light-cone gauge the operator $O_{\Delta G}(\Delta;0)$ in (5) reduces to:

$$2O_{\Delta G}(\Delta;0) = \partial^+A^2(\Delta)\partial^+A^1(0) - \partial^+A^1(\Delta)\partial^+A^2(0)\,, \quad (21)$$

where A^1 and A^2 denote the transverse components of the gluon field. This simplification occurs because in this gauge the path-ordered exponential $[\Delta;0]=1$, and the components of the gluon field strength tensor which enter $O_{\Delta G}(\Delta;0)$ are given by $G^{+\perp}=\partial^+A^\perp$. Using the definition (18) we obtain:

$$\begin{aligned}\Delta G(\mu^2) &= \frac{1}{2(s\cdot n)(p\cdot n)}\int_0^\infty dz\,\langle p,s|\partial^+A^2(\Delta)\partial^+A^1(0) \\ &\quad -\partial^+A^1(\Delta)\partial^+A^2(0)|p,s\rangle_{\mu^2}\,, \\ &= \frac{1}{2(s\cdot n)}\int_0^\infty d\lambda\,\langle p,s|\frac{\partial}{\partial\lambda}A^2(\lambda n)\partial^+A^1(0) \\ &\quad -\frac{\partial}{\partial\lambda}A^1(\lambda n)\partial^+A^2(0)|p,s\rangle_{\mu^2}. \end{aligned} \quad (22)$$

Assuming that the boundary term vanishes for $\lambda\to\infty$, one finally obtains:

$$2(s\cdot n)\Delta G(\mu^2) = \langle p,s|A^1(0)\partial^+A^2(0) - A^2(0)\partial^+A^1(0)|p,s\rangle, \quad (23)$$

which is easily shown to be equivalent to (20). This relation involves an operator which is built from local fields. Therefore its matrix element is calculable using known methods to deal with local operators. This fact however turns out to be of no real advantage as we will point out in Sect. 4, where we describe a QCD sum rule calculation.

To complete our discussion we derive the scale dependence of the gluon polarization $\Delta G(\mu^2)$ starting out from the operator definition (18). The one-loop evolution equation for the Ioffe-time distribution $\Gamma(z,\mu^2)$ reads [19,20]:

$$\mu^2\frac{d\Gamma(z,\mu^2)}{d\mu^2} = \frac{\alpha_s(\mu^2)}{2\pi}\int_0^1 dv[K_1(v)\Gamma(vz,\mu^2) + K_2(v)\frac{1}{z}\Sigma(vz,\mu^2)], \quad (24)$$

where:

$$\begin{aligned}K_1(v) &= \frac{\beta_0}{2}\delta(\bar v) + 2C_A\left(2v\bar v + \frac{1}{\bar v_+} - 1 - v\right), \\ K_2(v) &= -C_F\left(\delta(\bar v) - 2\bar v\right),\end{aligned} \quad (25)$$

with $C_F=4/3$, $C_A=3$, $\beta_0 = 11-\frac{2}{3}N_f$, with N_f being the number of active flavors, and $\bar v = 1-v$. The factor $1/z$ in (24) arises in accordance with the definition of $\Sigma(z,\mu^2)$, the Ioffe-time distribution of polarized quarks:

$$\begin{aligned}&\langle p,s|\bar\Psi(\Delta)\hat n\gamma_5[\Delta;0]\Psi(0)|p,s\rangle + (\Delta\to-\Delta) \\ &= 4(s\cdot n)\Sigma(z,\mu^2), \\ &\text{with}\quad \Sigma(z,\mu^2) = \int_0^1 du\,\Delta q(u,\mu^2)\cos(uz)\,. \end{aligned} \quad (26)$$

Here $\Delta q(u,\mu^2)$ is the flavor-singlet polarized quark distribution in momentum space. The evolution equation for $\Delta G(\mu^2)$ is obtained by integrating both sides of (24) over z. We find that gluons enter only via the term in K_1 being proportional to β_0. Furthermore the quark contribution can be transformed conveniently using the identity:

$$\frac{1}{z}\int_0^1 dv\,(\delta(\bar v) - 2\bar v)\cos(uvz) = -2u\int_0^1 dvv\left(1-\frac{1}{2}v\right)\sin(uvz). \quad (27)$$

Applying (17) yields then the standard one-loop evolution equation [18]:

$$\frac{d\Delta G(\mu^2)}{dt} = \frac{\alpha_s}{2\pi}\left(\frac{\beta_0}{2}\Delta G(\mu^2) + \frac{3}{2}C_F\Sigma(\mu^2)\right), \quad (28)$$

$$\text{with}\quad t=\log(\mu^2),\ \text{and}\ \alpha_s = \frac{4\pi}{\beta_0 t}.$$

Here

$$\Sigma(\mu^2) = \int_0^1 du\,\Delta q(u,\mu^2) \quad (29)$$

is the quark polarization in the proton[3]. At the one loop level one has [18]:

$$\frac{d\Sigma(\mu^2)}{dt} = 0, \quad \text{and thus}\quad \Sigma(\mu^2) = \Sigma_0 = \text{constant}, \quad (30)$$

i.e. to this accuracy the quark polarization is scale-independent. One then obtains as a solution of (28) the well known result [18,21]:

$$\Delta G(\mu'^2) = \frac{\alpha_s(\mu^2)}{\alpha_s(\mu'^2)}\Delta G(\mu^2) + \frac{4}{\beta_0}\Sigma_0\left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu'^2)} - 1\right). \quad (31)$$

Thus we have shown that the operator definition of $\Delta G(\mu^2)$ is indeed equivalent to parton model considerations.

3 Different gluon polarization partonometers

As discussed above, the gluon polarization in the nucleon can be obtained either from the integral over the Ioffe-time distribution (18), or in light-cone gauge from the forward matrix element of the topological current (20). Here we explore and compare both approaches in the framework of QCD sum rules.

[3] For considerations of the one-loop evolution equation for $\Delta G(\mu^2)$ it is not necessary to discuss the role of the axial anomaly in the interpretation of $\Sigma(\mu^2)$

3.1 Gluon polarization from the topological current

To estimate the nucleon matrix element of the topological current (20) we perform a standard QCD sum rule calculation. For this purpose we consider the three-point correlation function

$$I_K = i^2 \int d^4x \, e^{ip\cdot x} \int d^4y \, e^{iq\cdot y} \langle 0|T[\eta_G(x) \, n\cdot K(y) \, \bar{\eta}_G(0)]|0\rangle \quad (32)$$

of the operator $n \cdot K$ in light-cone gauge and the nucleon interpolating currents η_G, $\bar{\eta}_G$. For the latter we take:

$$\eta_G(x) = \frac{2}{3}(\eta_{\rm old}(x) - \eta_{\rm ex}(x)), \quad (33)$$
$$\eta_{\rm old}(x) = \epsilon^{abc}(u^{aT}(x)C\gamma_\mu u^b(x))\gamma_5\gamma^\mu\sigma_{\alpha\beta}\left[gG^{\alpha\beta}(x)d(x)\right]^c,$$
$$\eta_{\rm ex}(x) = \epsilon^{abc}(u^{aT}(x)C\gamma_\mu d^b(x))\gamma_5\gamma^\mu\sigma_{\alpha\beta}\left[gG^{\alpha\beta}(x)u(x)\right]^c.$$

Its overlap with the nucleon state,

$$\langle 0|\eta_G(0)|p,s\rangle = m_N^2\lambda_G u(p,s)\,, \quad (34)$$

at the scale $\mu^2 \sim 1$ GeV^2 has been determined in [22]. Note that this current, which contains explicit gluon degrees of freedom, has been successfully employed in investigations of nucleon matrix elements of QCD operators being sensitive to gluon components of the nucleon wave function [22–24]. To avoid large t-channel contributions we stay at Euclidean momenta $Q^2 = -q^2 \approx (1-4)$ GeV^2 and perform a numerical extrapolation to $Q^2 = 0$ at the end. Furthermore the kinematic is chosen such that $q \cdot n = 0$, i.e. $q^2 = -\mathbf{q}_\perp{}^2$.

In the following we concentrate on the contribution to the correlator (32) of the form:[4]

$$\gamma_5 \, \hat{p} \, (p\cdot n) T_K(p^2,(p+q)^2,Q^2)\,. \quad (35)$$

The invariant function T_K can be projected out uniquely from I_K. It receives contributions from nucleons as well as from higher resonances and continuum states. For the nucleon contribution we have:

$$T_K(p^2,(p+q)^2,Q^2) = \frac{{\rm Tr}\,(\hat{n}\gamma_5 I_K)}{4(p\cdot n)^2} = \frac{\lambda_G^2 \, m_N^4}{(m_N^2-p^2)(m_N^2-(p+q)^2)}\frac{\alpha_s}{\pi}\Delta\tilde{G}(Q^2), \quad (36)$$

where the form factor $\Delta\tilde{G}(Q^2)$ coincides at $Q^2 = 0$ with the gluon polarization ΔG.

As a next step we use the fact that T_K admits a double spectral representation [25]:

$$T_K(p^2,(p+q)^2,Q^2) = \int \frac{ds_1}{s_1 - p^2}\int \frac{ds_2}{s_2-(p+q)^2} \times \rho_K(s_1,s_2,Q^2). \quad (37)$$

[4] We suppress here and in the following any dependence on the scale μ^2

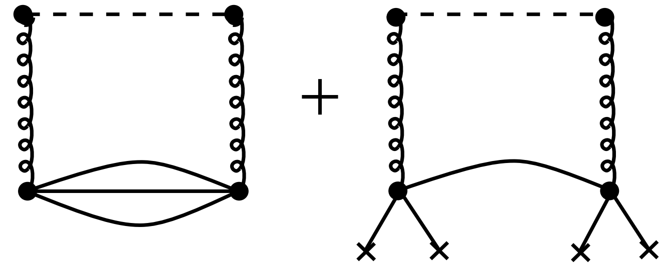

Fig. 1. Graphs representing the contributions from dimension-1 and dimension-6 operators to the QCD sum rule calculations described in the text

We have calculated the spectral density ρ_K in light-cone gauge, taking into account the dimension-1 and dimension-6 operators in the OPE of T_K, which are visualized in Fig. 1. We obtain:

$$\rho_K^{(1)}(s_1,s_2,Q^2) = \frac{\alpha_s^2\, Q^2 \left(\Delta_q - R^{\frac{1}{2}}\right)^2}{23040\,\pi^6\, R^{\frac{5}{2}}} \times\Big(\Delta_q\, Q^2\, R^{\frac{3}{2}} - 3\,\Delta_q\, R^2 - Q^2\, R^2 -7\, R^{\frac{5}{2}} - 2\,\Delta_q\, Q^2\, R^{\frac{1}{2}}\, s_1\, s_2 + 2\,\Delta_q\, R\, s_1\, s_2 +4\, R^{\frac{3}{2}}\, s_1\, s_2 - 4\, Q^2\, s_1^2\, s_2^2\Big)\,,$$
$$\rho_K^{(6)}(s_1,s_2,Q^2) = \frac{56\,\alpha_s^2}{9\,\pi^2}\,\langle\bar{q}q\rangle^2\,\frac{\Delta_q\, Q^4\, s_1\, s_2}{R^{\frac{5}{2}}}, \quad (38)$$

where $\Delta_q = s_1 + s_2 + Q^2$ and $R = \Delta_q^2 - 4s_1s_2$. To eliminate contributions from higher resonances and the continuum we limit the integral over s_1 and s_2 by the continuum threshold s_0. Performing in addition a Borel transformation in p^2 and $(p+q)^2$ yields the sum rule:

$$\frac{\alpha_s}{\pi}\Delta\tilde{G}(Q^2) = \frac{e^{m_N^2/M^2}}{\lambda_G^2 m_N^4}\int_0^{s_0} ds_1 \int_0^{s_0} ds_2\, e^{-(s_1+s_2)/2M^2} \times\rho_K(s_1,s_2,Q^2)\,, \quad (39)$$

where M^2 is the Borel parameter. For consistency both, s_0 and M^2 should be taken around their values fixed by the two-point sum rules [22]. In the actual calculation we use at the scale $\mu^2 = 1\,{\rm GeV}^2$ the standard value for the quark condensate, $-(2\pi)^2\langle\bar{q}q\rangle = 0.67\,{\rm GeV}^3$, and the strong coupling $\alpha_s = 0.37$ [22].

The stability of the sum rule (39) against variations of the Borel parameter M^2 and the continuum threshold s_0 is illustrated in Figs. 2 and 3. One observes a relatively strong dependence on s_0 which can be traced back to the large dimension of the interpolating current η_G. To determine the gluon polarization ΔG we extrapolate the form factor $\Delta\tilde{G}(Q^2)$ to $Q^2 \to 0$. For this purpose we fit the RHS of (39) in the interval $1\,{\rm GeV}^2 \le Q^2 < 4\,{\rm GeV}^2$ by:

$$\Delta\tilde{G}(Q^2) = \Delta G\,\frac{1}{[1+Q^2/m^2]^\gamma}\,, \quad (40)$$

with $\gamma = 3$ as suggested by quark counting rules. Extrapolating to $Q^2 \to 0$ gives $\Delta G(\mu^2 \sim 1\,{\rm GeV}^2) = 0.6 \pm 0.2$,

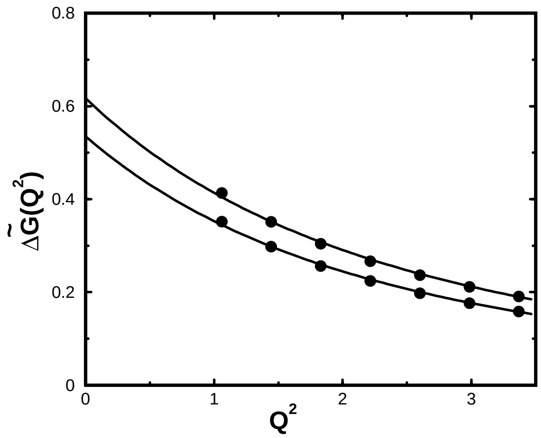

Fig. 2. Stability of the sum rule for the nucleon matrix element of $n \cdot K$ against variations of the Borel mass between $M^2 = 1\,\mathrm{GeV}^2$ (*lower curve*) and $M^2 = 2\,\mathrm{GeV}^2$ (*upper curve*). The continuum threshold has been fixed at $\sqrt{s_0} = 1.5\,\mathrm{GeV}$. The *dots* correspond to the sum rule results (39) for $1\,\mathrm{GeV}^2 \leq Q^2 < 4\,\mathrm{GeV}^2$. The *solid lines* show the extrapolation to $Q^2 \to 0$ (40)

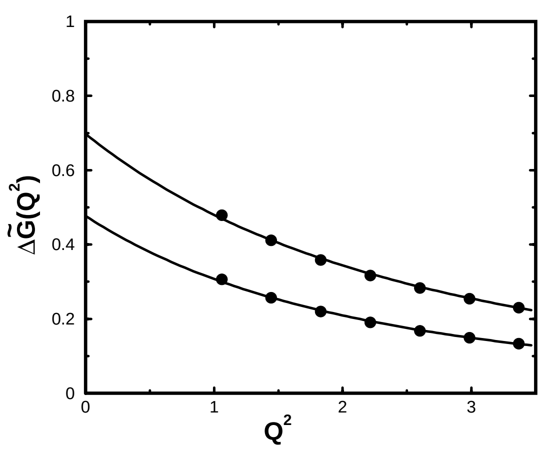

Fig. 3. Stability of the sum rule for the nucleon matrix element of $n \cdot K$ against variations of the continuum threshold between $\sqrt{s_0} = 1.4\,\mathrm{GeV}$ (*lower curve*) and $\sqrt{s_0} = 1.6\,\mathrm{GeV}$ (*upper curve*). The Borel mass has been fixed at $M^2 = 1.5$ GeV^2. The *dots* correspond to the sum rule results (39) for $1\,\mathrm{GeV}^2 \leq Q^2 < 4\,\mathrm{GeV}^2$. The *solid lines* show the extrapolation to $Q^2 \to 0$ (40)

where the error has been estimated from the M^2 and s_0 dependence of the extrapolation. We have checked that this result essentially does not change if we allow γ as a fit parameter as well. An additional 30% error arises from the uncertainty in λ_G and the vacuum saturation ansatz for the four-quark condensate [22].

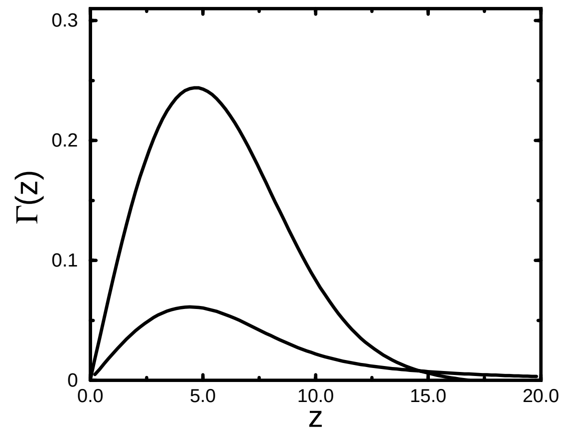

Fig. 4. The Ioffe-time distributions corresponding to the polarized gluon distributions of Chiappetta et al. [29] (*upper curve*) and and Brodsky et al. [27] (*lower curve*)

3.2 Gluon polarization from an approximate Ioffe-time distribution

In comparison to the above calculation we review here an estimate of ΔG via an approximate Ioffe-time distribution [16]. The latter is based on the conjecture that in the laboratory frame the polarized gluon distribution (18) receives major contributions only from longitudinal distances smaller than the nucleon diameter, as determined by the nucleon electromagnetic form factor. This hypothesis is supported both by Regge phenomenology [26] and the color coherence hypothesis [27], which impose strong restrictions on polarized glue at small u or, equivalently, at large z. For a more accurate determination of the longitudinal length scale at which the gluon polarization (18) effectively saturates an additional assumption has to be made. In this respect we have constructed in [16] a polarized gluon correlation function $\Gamma(z)$ which behaves at large z similar to the valence quark distribution of a nucleon, i.e. it becomes small for $z \gtrsim 10$. (In the nucleon rest frame $z = 10$ corresponds to a longitudinal distance of 2 fm.) Indeed most of the current parametrizations for polarized glue sustain this picture [28], as shown in Fig. 4 for the distributions from [27,29].

In Fig. 5 we present the Ioffe-time distribution as obtained from the expansion in (19), taking into account terms up to the n-th moment of $\Delta G(u)$. One finds that the first two non-vanishing moments Γ_3 and Γ_5 determine the Ioffe-time distribution $\Gamma(z)$ at small z nearly up to its maximum[5]. Therefore if the polarized gluon distribution is of regular shape similar to the ones shown in Fig. 4, and obtains significant contributions only from the region $z \lesssim 10$, a simple estimate of $\Delta G(\mu^2)$ is possible [16]. It is given by the area of the triangle spanned by the points $z = 0$, $z = 10$, and the maximum of the approximate

[5] Note that in the notation of [16] Γ_3 and Γ_5 correspond to Γ_2 and Γ_4, respectively

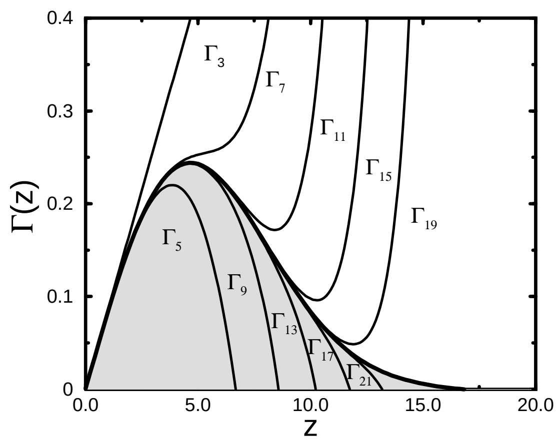

Fig. 5. The Ioffe-time distribution of polarized glue, approximated by its n-th order Taylor expansion around $z = 0$ (19), using the parameterization of [29]

Ioffe-time distribution

$$\Gamma(z) \approx \Gamma_3\, z - \frac{1}{6}\,\Gamma_5\, z^3. \tag{41}$$

This rough estimate requires only the knowledge of two moments of $\Delta G(u)$. These can, as a matter of principle, be taken from any theoretical investigation, i.e. QCD sum rules or lattice calculations.

Of course one can do better if the normalization of $\Gamma(z)$ is known at some large value of $z \approx 10$. Then a more accurate estimate can be achieved, calculating the area bound by the approximate Ioffe-time distribution (41) up to its maximum and a straight line connecting this point with the value of $\Gamma(z = 10)$. For the model parametrizations [27,29] this leads to an estimate for ΔG with $(10 - 20)\%$ accuracy. In [16] the color coherence hypothesis [27] has been applied to obtain information on the large z or, equivalently, small u behavior of polarized glue in nucleons:

$$\frac{\Delta G(u)}{G(u)} \to u, \quad \text{for } u \to 0. \tag{42}$$

It allows to estimate $\Gamma(z = 10)$ from parametrizations for the unpolarized gluon distribution. For example one finds from the GRV [30] and CTEQ [31] LO unpolarized gluon distributions $\Gamma(z = 10, \mu^2 \sim 1\,\mathrm{GeV}^2) = 0.005 - 0.007$. On the other hand the parametrizations of [27] and [29] yield $\Gamma(z = 10, \mu^2 \sim 1\,\mathrm{GeV}^2) = 0.02$ and 0.09, respectively. Since in principle nothing prevents $\Gamma(z)$ from becoming negative at large z, $-0.05 \leq \Gamma(z = 10) \leq 0.05$ should be a conservative estimate.

The moments Γ_3 and Γ_5 have been calculated within a standard QCD sum rule approach, starting from an investigation of the three-point correlation function [16]:

$$\begin{aligned} I_\Gamma &= i^2 \int d^4x\, e^{iq\cdot x} \int d^4y\, e^{ip\cdot y} \\ &\quad \times \langle 0|T[\eta_G(x) O_{\Delta G}(y + \Delta/2; y - \Delta/2)\bar{\eta}_G(0)]|0\rangle, \\ &= \gamma_5\, \hat{p}\, (p\cdot n)^2\, T_\Gamma(p^2, (p+q)^2, Q^2, z) + \ldots. \end{aligned} \tag{43}$$

Separating the contribution from nucleon states yields:

$$\begin{aligned} T_\Gamma(p^2, (p+q)^2, Q^2, z) &= \frac{\mathrm{Tr}\,(\hat{n}\gamma_5 I_\Gamma)}{4(p\cdot n)^3} \\ &= \frac{\lambda_G^2\, m_N^4}{(m_N^2 - p^2)(m_N^2 - (p+q)^2)} \tilde{\Gamma}(z, Q^2) + \ldots, \end{aligned} \tag{44}$$

where the form factor $\tilde{\Gamma}(z, Q^2)$ coincides at $Q^2 = 0$ with the Ioffe-time distribution $\Gamma(z)$ in (16). In a next step again a double spectral representation is introduced:

$$\begin{aligned} T_\Gamma(p^2, (p+q)^2, Q^2, z) &= \int \frac{ds_1}{s_1 - p^2} \int \frac{ds_2}{s_2 - (p+q)^2} \\ &\quad \times \rho_\Gamma(s_1, s_2, Q^2, z). \end{aligned} \tag{45}$$

As in Sect. 3.1 the dimension-1 and dimension-6 contributions have been considered. For them the polarized spectral density ρ_Γ coincides with the spectral density for the operator (4) which determines the unpolarized gluon distribution $G(u, \mu^2)$ [22]. Since this spectral density leads to a realistic large value for the contribution of gluons to the nucleon momentum, it is suggestive that also the integrated gluon polarization is large.

Expanding (44,45) in powers of z yields sum rules for $\Gamma_3(Q^2)$ and $\Gamma_5(Q^2)$, which are given explicitly in [16]. An extrapolation to $Q^2 = 0$ as in (40) finally leads to the desired moments. In combination with the color coherence hypothesis (42) one then obtains $\Delta G(\mu^2 \sim 1\,\mathrm{GeV}^2) = 2 \pm 1$. The main source of the quoted error is due to uncertainties in the QCD sum rule approach. Furthermore, under the assumption that $\Gamma(z)$ follows the Regge behavior at large z, possible contributions from longitudinal distances beyond the ones accounted for have been estimated to be smaller than ± 0.2. Note however that current experience [32,33] indicates that QCD sum rules significantly overestimate higher moments of parton distribution functions. Therefore the result quoted above should be treated as an upper limit for $\Delta G(\mu^2 \sim 1\,\mathrm{GeV}^2)$.

4 Discussion

We have found that the gluon polarization obtained from the matrix element of the topological current differs by a factor of around four as compared to the estimate based on the approximate Ioffe-time distribution. This obviously calls for an explanation. As we show below, this discrepancy emphasizes the role played by contributions from different longitudinal distances.

First let us comment on the question whether we have gained anything by going from the non-local definition of ΔG to the local one. To this end note that due to (20) one has:[6]

$$\rho_K(s_1, s_2, Q^2) = \int_0^\infty dz\, \rho_\Gamma(s_1, s_2, Q^2, z)\,. \tag{46}$$

[6] We remind the reader that this relation holds only if the gauge-dependent density ρ_K is calculated in light-cone gauge

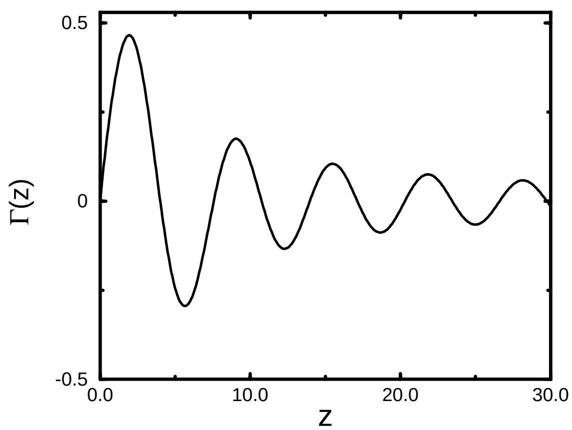

Fig. 6. The Ioffe-time distribution $\Gamma(z)$ of polarized glue from QCD sum rules for the leading dimension-6 contribution, taken in the limit $Q^2 \to 0$

Of course this relation can be directly verified for the contributions from the operators of dimension one and six as used in Sect. 3.1 and 3.2. This however implies that, in accordance with the equivalence of the definitions (18) and (20), the matrix element of $n \cdot K$ is as sensitive to different longitudinal distances as the matrix element of the nonlocal operator (5). Consequently the apparent local form of K does not remove the essential non-local character of ΔG.

To clarify the reason for our different results for ΔG consider a polarized gluon distribution with the following behavior:

$$G(u) \sim \begin{cases} \mathcal{A}\, u^{-\alpha}, & \text{for} \quad u \to 0, \\ \mathcal{B}\,(1-u)^{\beta}, & \text{for} \quad u \to 1\,. \end{cases} \tag{47}$$

The asymptotic form of the corresponding Ioffe-time distribution at large z arising from the regions $u \to 0$ and $u \to 1$ reads:

$$\begin{aligned} \Gamma(z) = {} & \mathcal{A} \sin\left(\frac{\pi}{2}\alpha\right) \Gamma_{\mathrm{E}}(2-\alpha) z^{\alpha-2} + \dots \\ & - \mathcal{B} \cos\left(z - \beta\frac{\pi}{2}\right) \Gamma_{\mathrm{E}}(\beta+1) z^{-1-\beta} \\ & + \dots, \end{aligned} \tag{48}$$

where Γ_{E} denotes the Euler Gamma function. Phenomenologically one would expect $0 < \alpha < 1$ and $\beta \geq 4$ [27], which guarantees a smooth non-oscillatory behavior of $\Gamma(z)$ at large distances. (In (48) even for $\alpha \sim 0$ the next-to-leading contribution from the small u region dominates over the leading one arising from the region $u \to 1$, if only β is large enough and $\mathcal{A}$ and $\mathcal{B}$ are of similar magnitude.)

The Ioffe-time distribution obtained from QCD sum rules behaves however differently. This can be seen from Fig. 6 where we show its dominant contribution which results from the dimension-6 operator taken in the limit $Q^2 \to 0$. This limit should give a qualitatively reasonable estimate since the Q^2-dependence of the sum rules in Sect. 3.1 and 3.2 has turned out to be quite smooth. We find that, contrary to what one expects from phenomenological considerations, $\Gamma(z)$ oscillates strongly and decreases relatively slow at large z. In terms of the characteristic exponents in (48) its behavior corresponds approximately to $\alpha \sim -1$ and $\beta \sim 0$. This shows that QCD sum rules yield a distribution skewed towards large values of u, and consequently leads to predictions for the moments of $\Delta G(u)$ which are too large. The small value of β reflects a large weight for configurations in which one gluon carries most of the momentum of the nucleon. In the present calculation this is due to the fact that neither a perturbative (Sudakov), nor a non-perturbative (large invariant mass) suppression mechanism for such configurations is present in lowest order OPE. Since the sum rule for $n \cdot K$ in Sect. 3.1 accounts for the full integral over $\Gamma(z)$ from zero to infinity, it incorporates all oscillations at intermediate and large z, leading to a small value of ΔG. Apart from the small-z region the z-dependence of $\Gamma(z)$ shown in Fig. 6 is certainly not realistic. Therefore we have to conclude that the QCD sum rule calculation of the matrix element of $n \cdot K$ is not entirely self-consistent, as it receives large contributions from regions where the sum rule method is not reliable.

This is different in the approach discussed in Sect. 3.2. Here we have assumed that QCD sum rules yield a reasonable estimate for the first two moments Γ_3 and Γ_5. However we have discarded oscillations of $\Gamma(z)$ at large z, assuming that the main contribution to ΔG arises from small light-cone distances. As a consequence QCD sum rules are used only in a domain where they are in principle applicable. We therefore believe that such an estimate of ΔG is better justified.

5 Summary and conclusions

In QCD the twist-2 polarized gluon distribution is defined through a gauge invariant but non-local string operator. As a consequence ΔG can receive, at least in principle, contributions from different longitudinal distances. Although in light-cone gauge ΔG can be formally expressed through the forward matrix element of the local topological current, also in this case contributions from all longitudinal distances are accumulated. Therefore the latter can be used for a trustworthy estimate of ΔG only if an approximation to strong interaction dynamics is available which is applicable at all longitudinal distances. We have illustrated this point in the framework of a QCD sum rule calculation which leads to rather unrealistic contributions from large longitudinal distances, and results in a small value for $\Delta G(\mu^2 \sim 1\,\mathrm{GeV}^2) = 0.6 \pm 0.2$.

For a self consistent estimate of the gluon polarization in nucleons one has to ensure that the main support to ΔG results from distances where the used approximation is supposed to do its best. In the case of QCD sum rules only contributions from small distances can be approximated in a reasonable way. Combining the latter with the assumption that contributions to ΔG from distances larger than the typical nucleon size are small yields $\Delta G(\mu^2 \sim 1\,\mathrm{GeV}^2) = 2 \pm 1$.

If, contrary to our assumption, ΔG receives at low normalization scales μ^2 important contributions from large longitudinal distances, new interesting questions would arise since such contributions could hardly be interpreted as being due to confining glue, understood as part of a nucleon with a size of around one fm. At large normalization scales GLAP evolution may result in a more and more singular behavior of $\Delta G(u, \mu^2)$ at small values of u. In this case it is understood, however, that such contributions have to be filtered out in order to learn something about the distribution of nucleon polarization among low-virtuality degrees of freedom.

To avoid in the calculation of moments of a parton distribution a situation in which an approximation tailored for contributions from small longitudinal distances generates large contributions from distances beyond its scope of applicability, one should require that it gives a reasonable behavior of the considered distribution at large values of z or, equivalently, at values of u close to 0[7]. If such a requirement cannot be fulfilled an explicit construction of contributions from large longitudinal distances , like in [16], is necessary.

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[7] To achieve this for example for the quark distribution in the bag model one has to introduce a complicated projection which allows to approximately express bag model solutions in terms of eigenfunctions of the momentum operator [34]

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